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Unit A – Exponential and Logarithmic Equations
PRACTICE TEST

PART A: – NO CALCULATOR
Complete the following section without the use of a calculator.

1. Rewrite $6^{-2} = \frac{1}{36}$ in logarithmic form.

$-2 = \log_6 \frac{1}{36}$

2. Rewrite $\log_3 81 = 4$ in exponential form.

$3^4 = 81$

2. Use the laws of logarithms to simplify and then evaluate each expression:

a) $\log_3 135 - \log_3 5$

$= \frac{\log_3 135}{\log_3 5}$ $x = 3$

$x = \log_3 27$

$5^x = 27$

b) $\log_7 \sqrt[5]{49}$

$x = \log_7 (49)^{1/5}$

$7^x = (49)^{1/5}$

$7^x = 7^{2/5}$

$\therefore x = \frac{2}{5}$

c) $\log_8 2 + 3\log_8 2 + \frac{1}{2}\log_8 16$

$\log_8 2 + \log_8 2^3 + \log_8 16^{1/2}$

$\log_8 (2 \cdot 8 \cdot 4)$

$\log_8 64$ $\log_8 8^2 = 2$

3. Write $\frac{1}{2}\left(4\log x - \log\sqrt{x^5}\right) + \log\sqrt[3]{x^2}$ as a single logarithm.

$\frac{1}{2}(\log x^4 - \log x^{5/2}) + \log x^{2/3}$

$\frac{1}{2}\log\left(\frac{x^4}{x^{5/2}}\right) + \log x^{2/3}$

$\log\left(x^{3/2}\right)^{1/2} + \log x^{2/3}$

$\log x^{3/4} + \log x^{2/3}$ $x^{3/4} \cdot x^{2/3}$

$\log x^{3/4 + 2/3}$ $\frac{9}{12} + \frac{8}{12}$

$\log x_{17/12}$

$\log x$

4. Solve algebraically.

a) $3^{3x-1} = \frac{1}{81}$

$3^{3x-1} = 3^{-4}$

$3x - 1 = -4$

$3x = -3$

$x = -1$

b) $64^{x-2} = 16^{4x}$

$(4^3)^{x-2} = (4^2)^{4x}$

$4^{3x-6} = 4^{8x}$

$3x - 6 = 8x$

$-6 = 5x$

$-6/5 = x$

5. Solve the equation $5^{3x-2} = 3^{x+5}$. Leave your answer as an exact value.

$\log 5^{3x-2} = \log 3^{x+5}$

$(3x-2)\log 5 = (x+5)\log 3$

$3x\log 5 - 2\log 5 = x\log 3 + 5\log 3$

$3x\log 5 - x\log 3 = 5\log 3 + 2\log 5$

$x = \frac{5\log 3 + 2\log 5}{3\log 5 - \log 3}$

6. Solve $\log_{11} x + \log_{11}(x+1) = \log_{11} 6$. Identify any extraneous roots.

$\log_{11}(x(x+1)) = \log_{11} 6$

$(x(x+1)) = 6$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, x = 2$

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Useful Formulas

Richter Scale	dB	PH
$M = \log\left(\frac{I}{I_0}\right)$	$L = 10\log(\frac{I}{I_o})$	$pH = -\log[H^+]$

$I_0 = 10^{-12}$

PART B: – CALCULATOR ALLOWED

1. Find the difference in pHs of a swimming pool with hydronium ion concentration of 4.4×10^{-8} mol/L and a swimming pool with hydronium ion concentration of 5.7×10^{-8} mol/L.

$ph_2 - ph_1 = -\log[4.4\times10^{-8}] - -\log[5.7\times10^{-8}]$

$= 7.357 - 7.244$

≈ 0.12

2. How much more intense is a 104 db sound than a 75 db sound?

$104 = 10\log\left(\frac{I}{10^{-12}}\right)$ $I = 10^{-1.6}$ $75 = 10\log\left(\frac{I}{10^{-12}}\right)$

$10.4 = \log\left(\frac{I}{10^{-12}}\right)$ $\frac{10^{-1.6}}{10^{-4.5}} = 10^{2.9}$ $7.5 = \log\left(\frac{I}{10^{-12}}\right)$

$10^{10.4} = \frac{I}{10^{-12}}$ $10^{7.5} = \frac{I}{10^{-12}}$ $I = 10^{-4.5}$

3. A chemist has a 20-mg sample of polonium-218. He needs exactly 81.5% of it for an experiment. Given that the half-life of polonium-218 is approximately 3.1 min, how long will it take for the sample to decay to the desired mass?

$N = N_0\left(\frac{1}{2}\right)^{t/h}$ $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/h}$

$16.3 = 20\left(\frac{1}{2}\right)^{t/3.1}$ $\frac{0.815N_0}{N_0} = \left(\frac{1}{2}\right)^{t/h}$

$0.815 = \left(\frac{1}{2}\right)^{t/3.1}$

$\log 0.815 = \frac{t}{3.1}\log\frac{1}{2}$

$t = 0.914$

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4. Given $f(x) = -2\log[(x-1)]+1$.

a) Identify the transformations.

- reflection in x-axis
- vertical stretch by a factor 2
- horizontal translation right 1
- up 1

b) Determine the x-intercept.

$0 = -2\log(x-1) + 1$

$-1 = -2\log(x-1)$

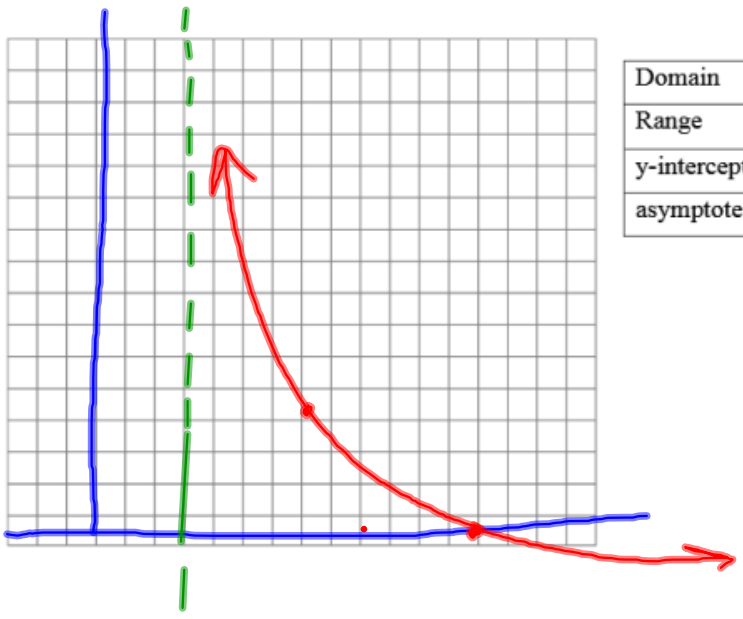
$\frac{1}{2} = \log(x-1)$

$10^{\frac{1}{2}} = x-1$

$3.16+1 = x$

$4.16 = x$

c) Sketch the function and determine the key features.



Domain	
Range	
y-intercept	None
asymptote	V.A. $x=1$

$y = \log 2x$

$y = \log 3x$

$y = \log \frac{1}{4}x$

5. Solve the equation $2(5^{2x}) - 16(5^x) + 30 = 0$.

$2(5^x)^2$

let $5^x = y$

$2y^2 - 16y + 30 = 0$

$y^2 - 8y + 15 = 0$

$(y-5)(y-3) = 0$

$y = 5$ or $y = 3$

$5^x = 5$ $\quad 5^x = 3$

$x = 1$ $\quad x = \frac{\log 3}{\log 5}$

6. Solve the equation $\sqrt[3]{256^2} \times 16^x = 64^{x-3}$.

$\sqrt[3]{(4^4)^2} \times (4^2)^x = (4^3)^{x-3}$

$(4^4)^{2/3} \times 4^{2x} = 4^{3x-9}$

$4^{8/3+2x} = 4^{3x-9}$

$\frac{8}{3} + 2x = 3x - 9$

$\frac{8}{3} + 9 = x$

$\frac{35}{3} = x$

7. Consider the expression $y = \log_2 a$.
   - For what values of "a" will this expression yield positive numbers, explain
   - For what values of "a" will this expression yield negative numbers, explain
   - For values of "a" will this expression be undefined, explain